# Experiments and nonlocal continuum modeling of the size-dependent fracture in elastomers[1,2]

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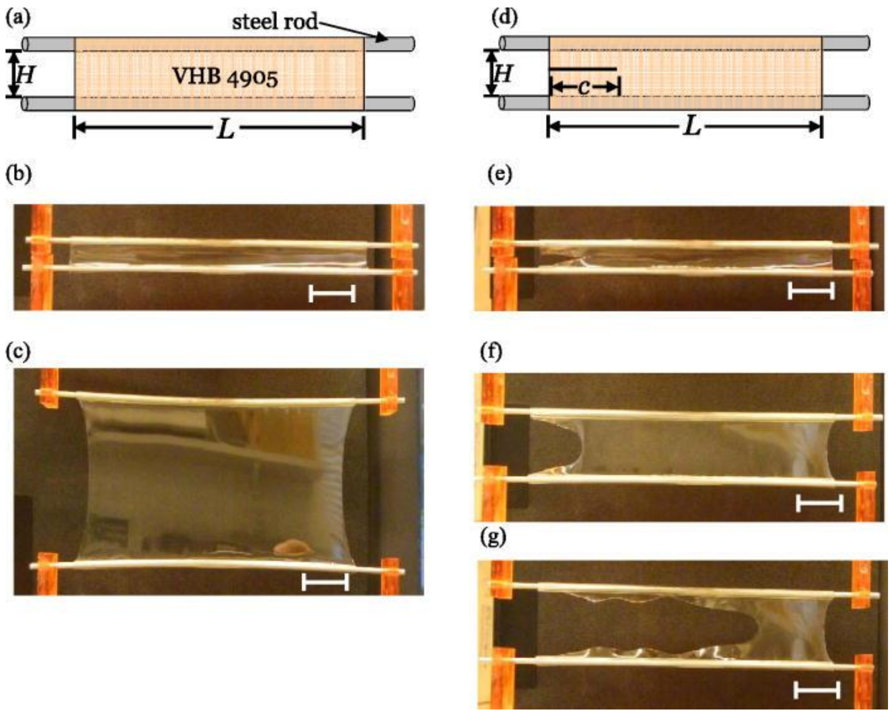
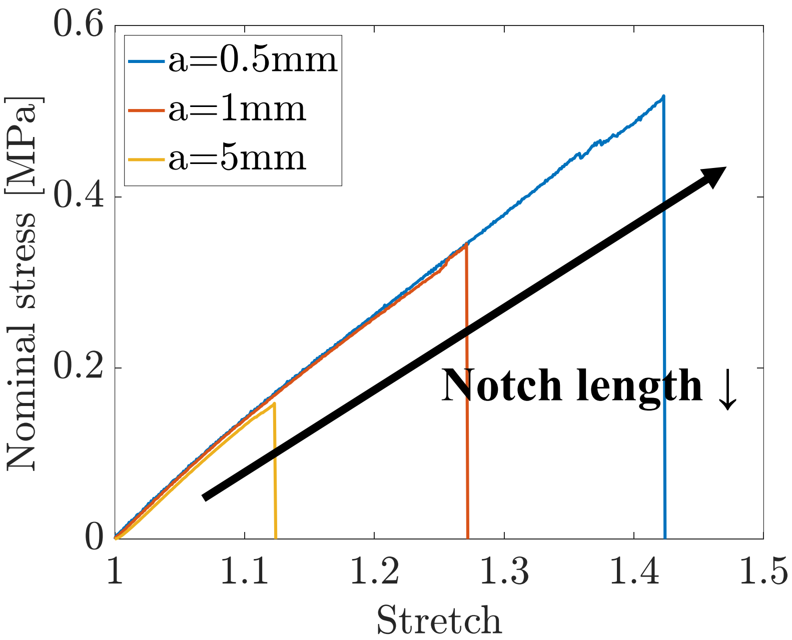
# Fracture in elastomers

- Extreme, nonlinear deformation → fracture
- Influenced by the size of flaws; **the size-dependent fracture**[1,3]
  - Rupture stretch increases as the specimen size decreases




a) The presence of flaws impacts the fracture behavior[4]



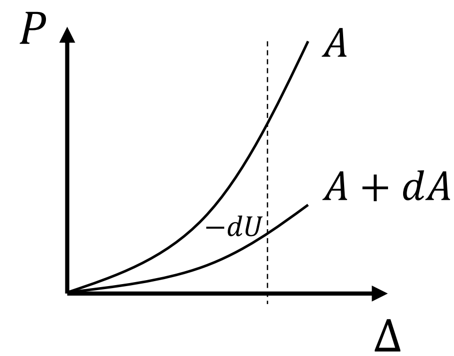

b) Size-dependent fracture in polydimethylsiloxane (PDMS) specimens

# Fracture in elastomers

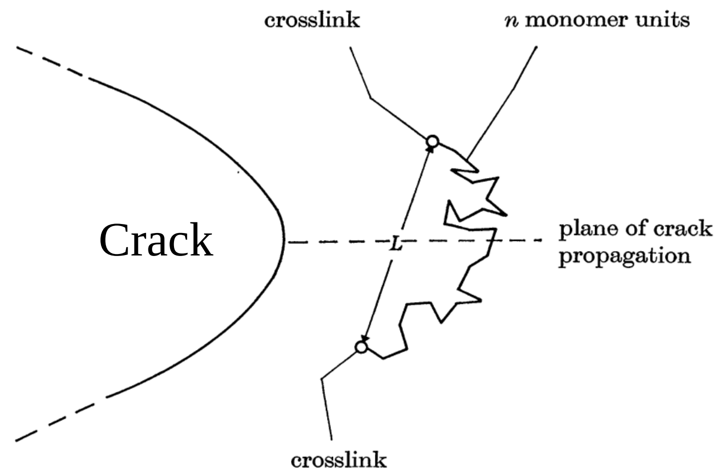


- Occurs when …
  - Macroscopically, **G reaches Γ**
    - Griffith theory[5,6]
    - G: Energy release rate
    - Γ: Fracture energy

$$G = -\frac{dU(\mathrm{P}, \Delta)}{dA}$$




  - Microscopically, $\boldsymbol{\varepsilon_R}$ **reaches** $\boldsymbol{\varepsilon_R^f}$
    - Lake-Thomas theory[7-9]
    - $\varepsilon_R$: Internal energy
    - $\varepsilon_R^f$: critical internal energy;
      bond dissociation energy




- These approaches are compatible (Lake and Thomas [7])

# Objectives



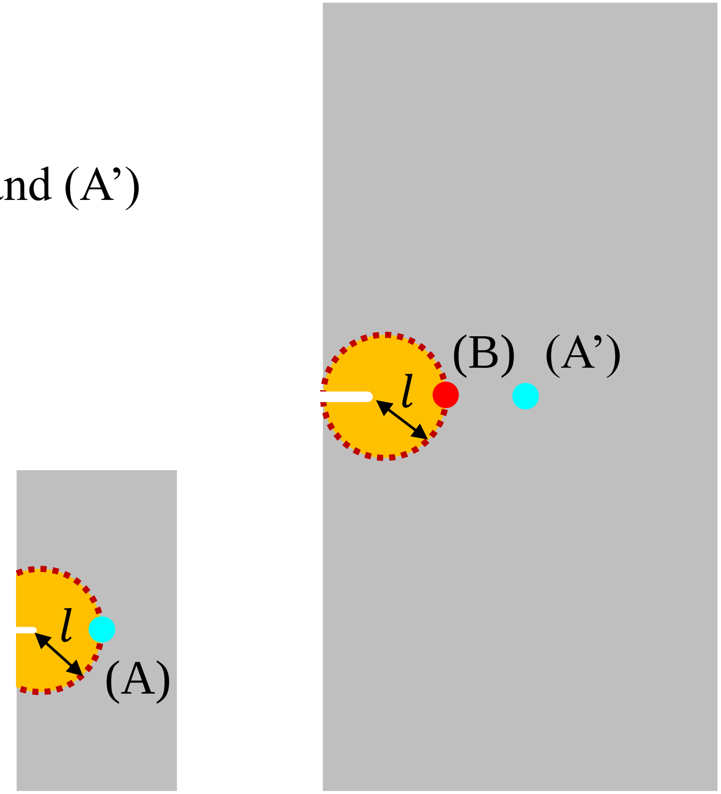
- Predicting the **size-dependent fracture** in elastomers[1]
    - Experiments and numerical simulations[2] were carried out
- **Internal energy-driven** fracture criterion; inspired by the Lake-Thomas model[7-9]

- Using the **phase-field model** rooted in the gradient-damage theory[2,9-13]
    - Mesh-insensitive crack propagation process
    - The internal energy-driven fracture criterion
    - Thermodynamics of the damage and fracture

# Size-dependent fracture & Fracture process zone

- Fracture process zone
  - Where the polymer chains rupture = Where the dissipation mainly occurs

- Stress at point (B) is larger than those at (A) and (A')
  - $\sigma_A = \sigma_{A'} < \sigma_B$
- → Free energy at point (B) is larger than those at (A) and (A')
  - $\psi_A = \psi_{A'} < \psi_B$
- → **$\boldsymbol{\psi_B}$ reaches the critical energy earlier than $\boldsymbol{\psi_A}$**
- → **The larger specimen ruptures earlier**




The size of fracture process zone[1,3,14,15]:

$$l = \frac{\Gamma}{W^*} = \frac{Fracture\ energy}{Critical\ deformation\ energy}$$

# Nonlocal continuum modeling

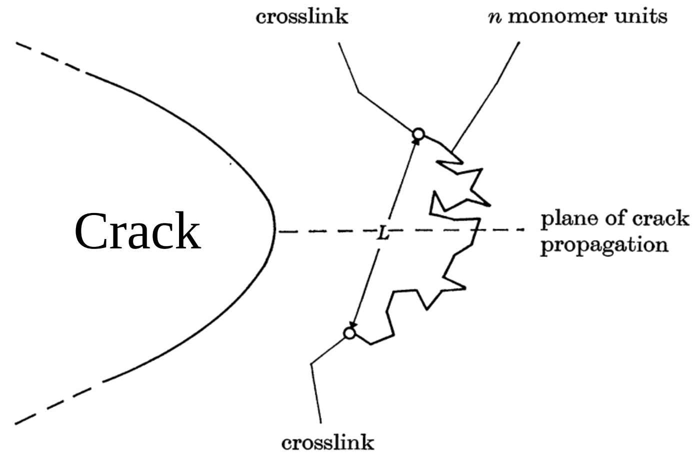
- 1. The damage $d \in [0,1]$
  - d=0: intact
  - d=1: fully damaged
- **Internal energy-driven** fracture criterion
  - Inspired by the Lake-Thomas model[5]
  - Fracture = **Scission of polymer chains**
- Governing equations[9]
  - Macroforce balance Div $\mathbf{T}_R = 0$
  - Microforce balance $\zeta \dot{d} = 2(1-d)\mathcal{H}_R - \hat{\varepsilon}_R^f (d - l'^2 \Delta d)$

History function;
the fracture criterion

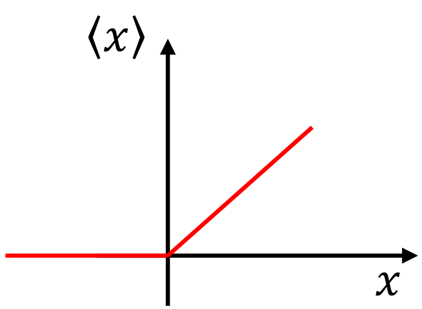
$$\mathcal{H}_R = \left\langle \varepsilon_R^0 - \varepsilon_R^f / 2 \right\rangle, \quad \text{where } \langle x \rangle = \begin{cases} x & if\ x > 0 \\ 0 & if\ x \leq 0 \end{cases}$$

Internal energy

a)[7]




b)



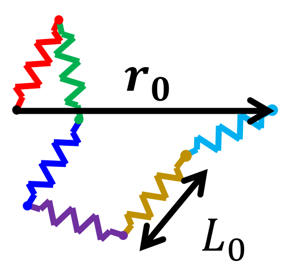
# Nonlocal continuum modeling



- **Internal energy** should be considered → **Bond stretch**[8,9]
  - Deformation = Chain configuration change + stretching of molecular bonds
- $\psi_R = (1-d)^2\left[\frac{1}{2}NnE_b(\lambda_b-1)^2+\frac{1}{2}K(J-1)^2\right] + Nk_b\theta n\left[\frac{\bar{\lambda}\lambda_b^{-1}}{\sqrt{n}}\beta+\ln\left(\frac{\beta}{\sinh\beta}\right)\right]+\frac{1}{2}\varepsilon_R^f l^2|\nabla d|^2$

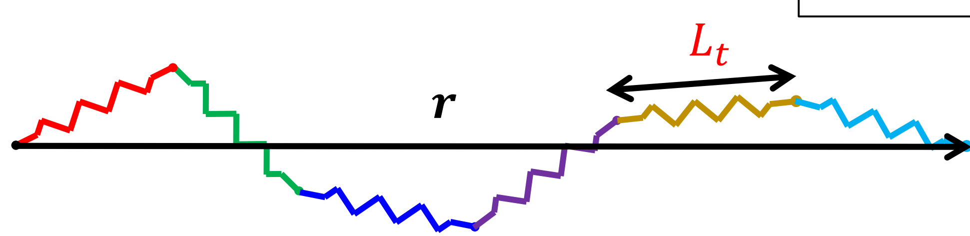
$(1-d)^2\varepsilon_R^0$; **Damage acts on the internal energy only** $-\theta\eta_R$; Entropic energy Nonlocal energy[9]

a) Reference configuration




$$\bar{\lambda} = \frac{|\boldsymbol{r}|}{|\boldsymbol{r}_0|}$$
$$\lambda_b = \frac{L_t}{L_0}$$
$$\mathcal{L}(x) = \coth x - \frac{1}{x}$$
$$\beta = \mathcal{L}^{-1}\left(\frac{\bar{\lambda}\lambda_b^{-1}}{\sqrt{n}}\right)$$

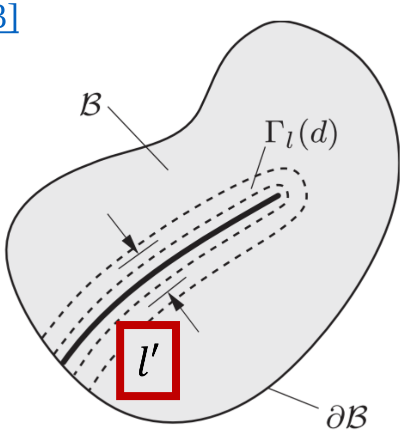
a) Deformed configuration



# Nonlocal continuum modeling

- 2. Phase-field model rooted in the gradient-damage theory[9-13]
  - "Diffusive damage zone"

a)[13]



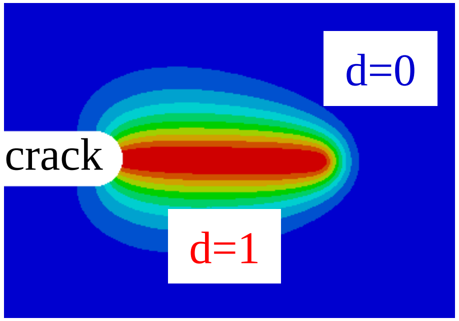

**Intrinsic length scale $l'$**

- Microforce balance $$\zeta \dot{d} = 2(1-d)\underline{\mathcal{H}_R} - \hat{\varepsilon}_{\mathrm{R}}^{f}(d - \boxed{l'^2 \Delta d})$$

History function;
the fracture criterion

- The **intrinsic length scale $l'$** → the size of diffusive damage zone
  - A numerical parameter; ambiguous physical meaning

b)




Crack propagation;
at reference configuration

# Nonlocal continuum modeling

- Assumption[1]: Diffusive damage zone = Fracture process zone
    - Regions of the damage evolution and the dissipation

- The size of fracture process zone

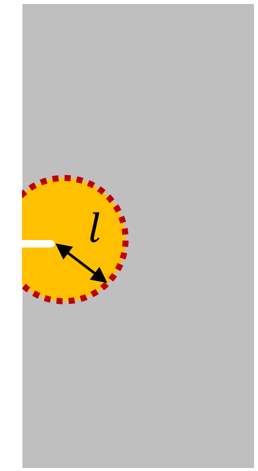
$$= \frac{\Gamma}{W^*} = \frac{Fracture\ energy}{Critical\ deformation\ energy}$$ → **Intrinsic length scale**

→ Identify the intrinsic length scale $l$ **from experiments**

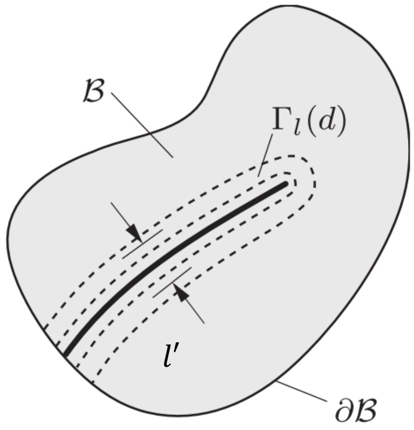
→ Apply to the phase field model

→ Predict the **size-dependent fracture** by numerical simulations[1]

a) Fracture process zone




b)[13] Diffusive damage zone



# Experimental procedures[1]



- Geometries
  - a = {0.5, 1, 5} mm
  - w = 10a,  h = 20a, specimen thickness: 0.5mm

    → w = {5, 10, 50} mm

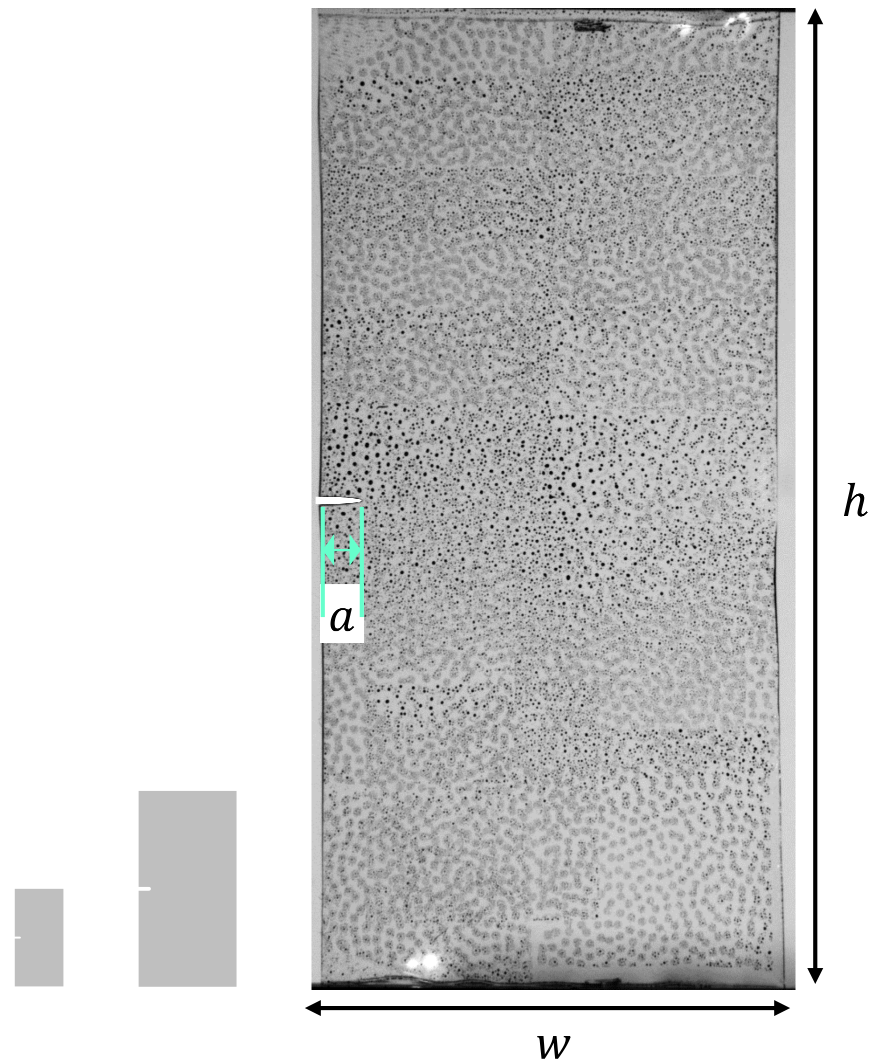
    → h = {10, 20, 100} mm

- Materials
  - PDMS
  - TangoPlus (3D-printed elastomer)

- Strain rate 0.01 $s^{-1}$, temperature ~21℃

- Digital image correlation (DIC) analysis

  → Strain fields from experiments



# The intrinsic length scale $l$



- $l = \frac{\Gamma}{W^*}$ → **Experimentally identified intrinsic length scale**[1]
- $\Gamma$: Fracture energy
  - from notched specimens
- $W^*$: Critical deformation energy
  - from unnotched specimens

**PDMS**

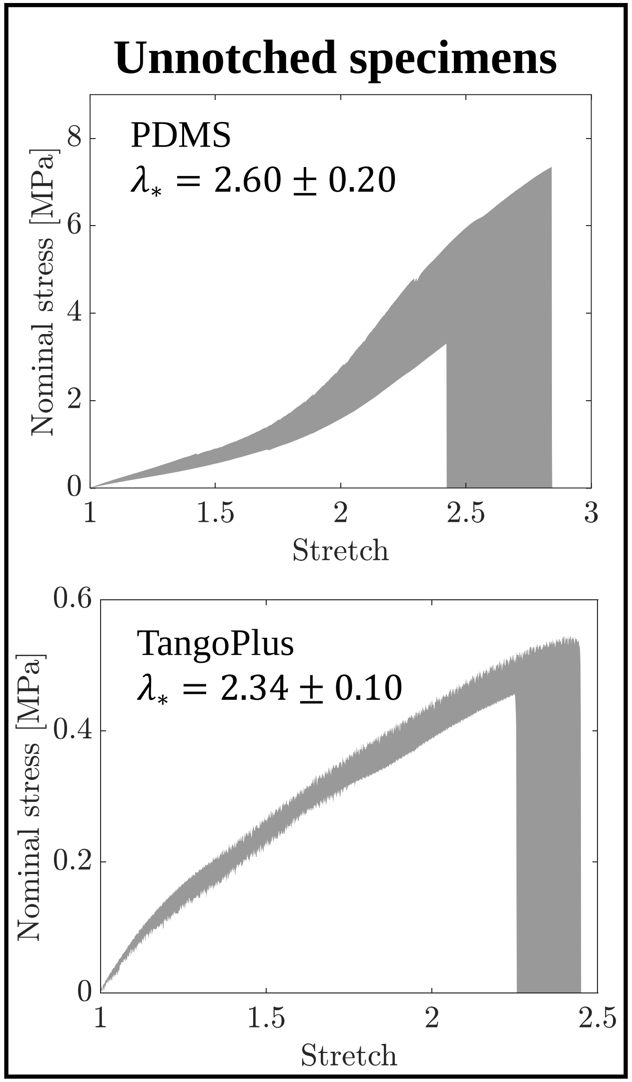
$\Gamma \approx 0.25\text{mJ/mm}^2$, $W^* \approx 2.7\text{mJ/mm}^3$

→ $\boldsymbol{l \approx 0.08mm}$

**TangoPlus**

$\Gamma = 0.5\text{mJ/mm}^2$, $W^* \approx 0.45\text{mJ/mm}^3$

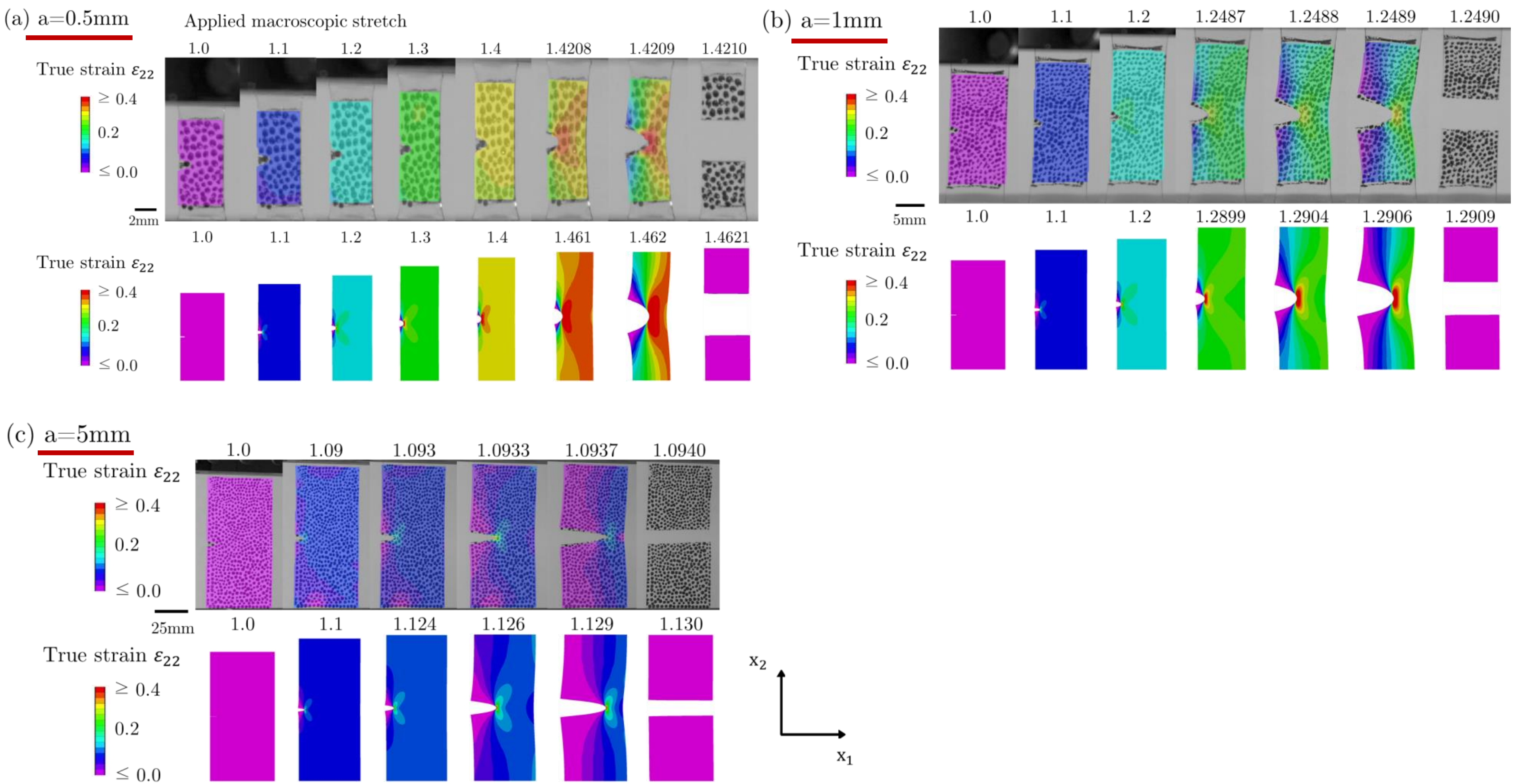
→ $\boldsymbol{l \approx 1mm}$



# Results: Experiment vs. Numerical simulation[1]

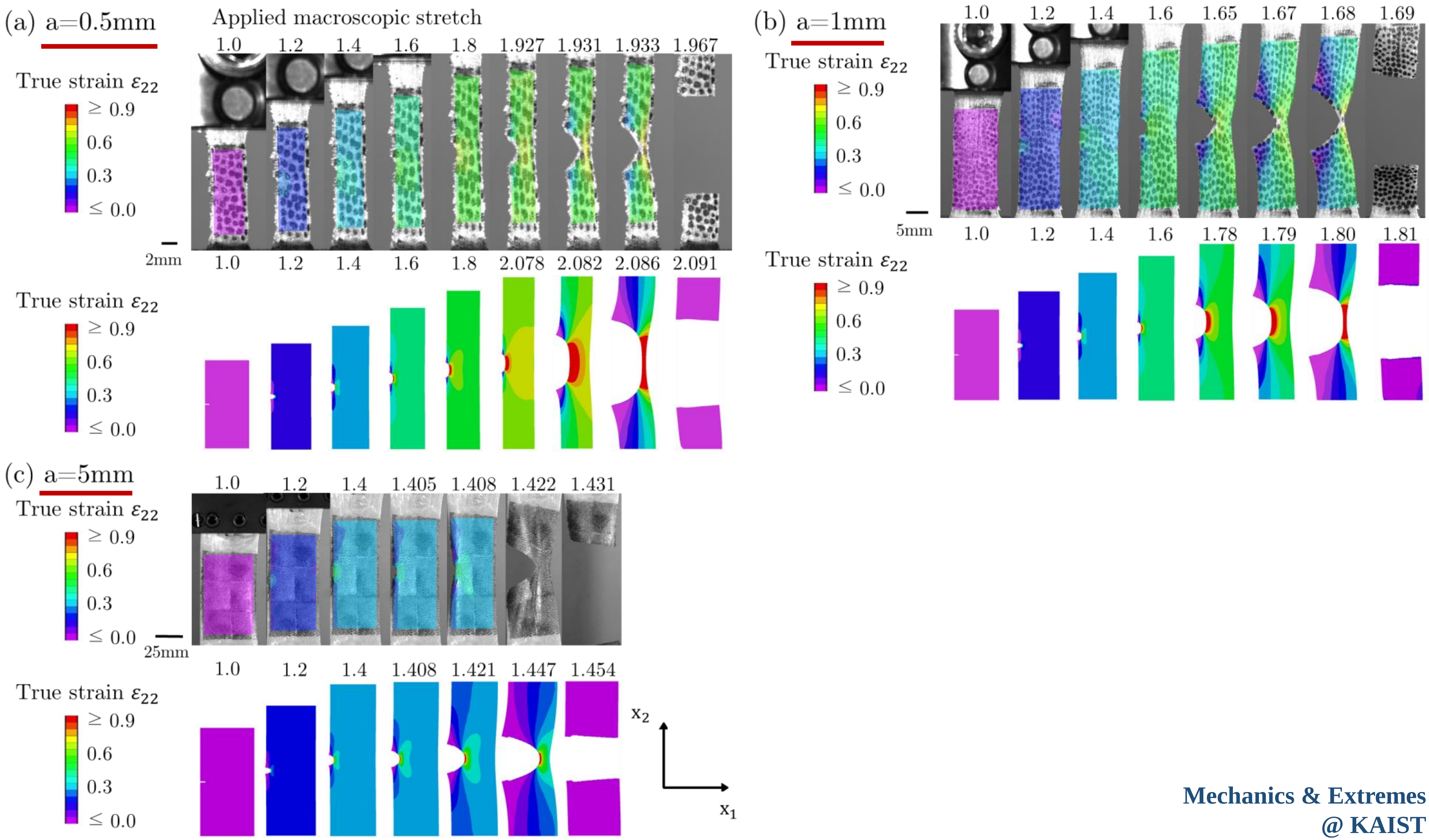


- Strain fields in **PDMS** specimens ($l = 0.08mm$)
  - Larger specimen ruptures earlier



# Results: Experiment vs. Numerical simulation[1]

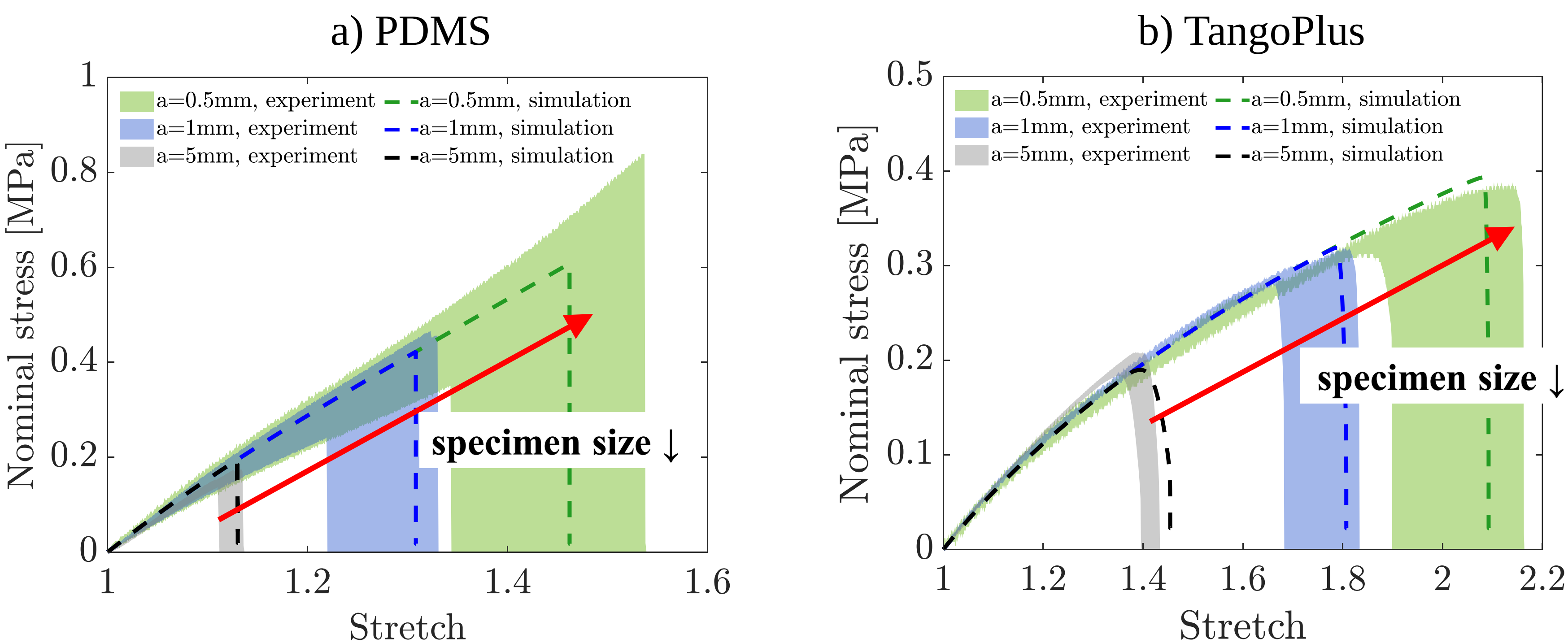
- Strain fields in **TangoPlus** specimens ($l = 1mm$)
    - Larger specimen ruptures earlier



# Results: Experiment vs. Numerical simulation[1]



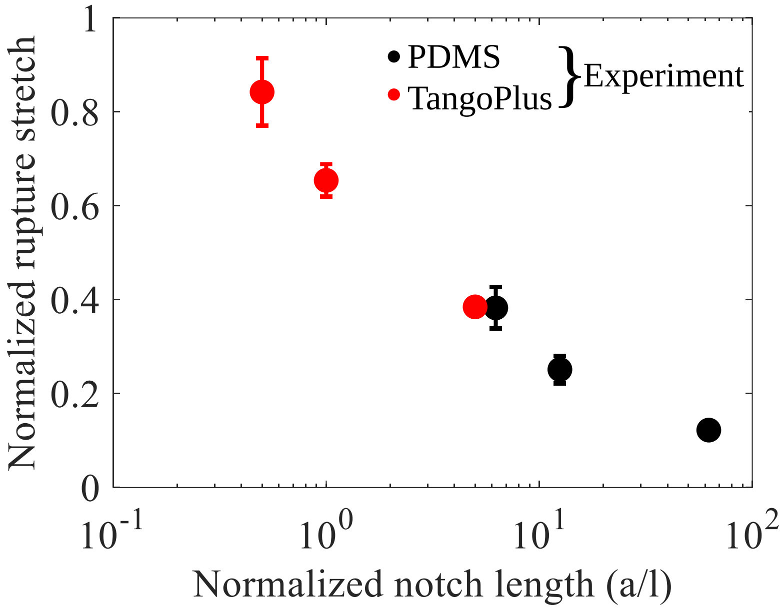
- Notch lengths $a$ = {0.5, 1, 5} mm
- Geometric similarity → **Identical initial stress-stretch response**
- Smaller notch length → Higher rupture stretch



# Notch-length sensitivity[1]

- PDMS vs. TangoPlus; same specimen sizes
  - PDMS: $l = 0.08mm$
  - TangoPlus: $l = 1mm$

**More than 10 times**




- Normalized rupture stretch

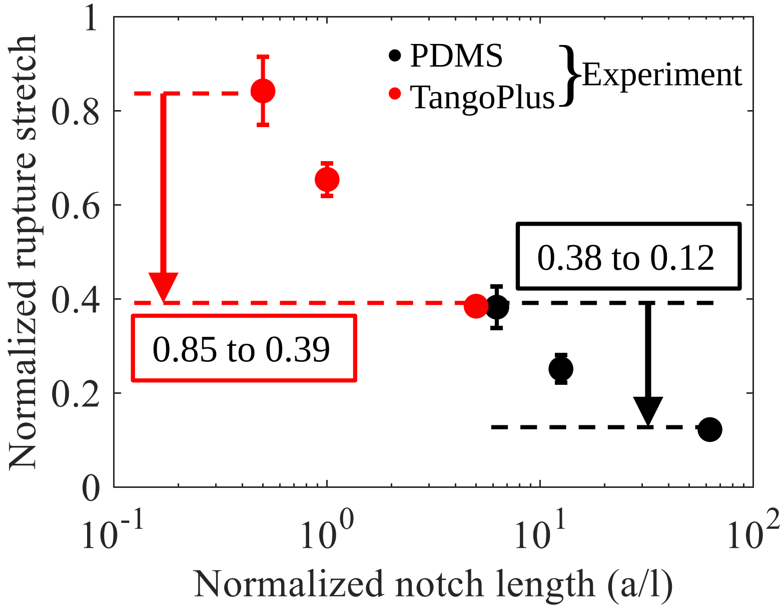
$$= \frac{\text{Rupture stretch of } \textbf{notched} \text{ specimens}}{\text{Rupture stretch of } \textbf{unnotched} \text{ specimens}}$$

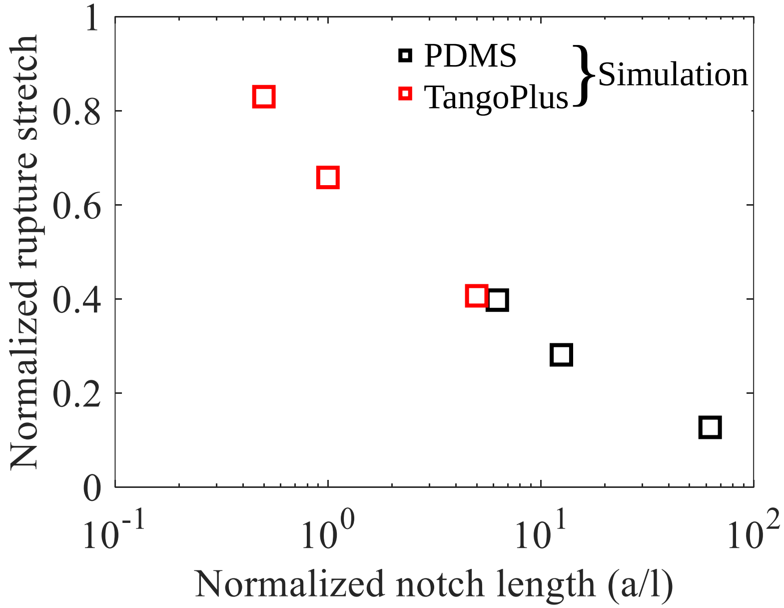
- Normalized notch length $= \frac{\text{Notch length (a)}}{\text{Intrinsic length scale } (l)}$

# Notch-length sensitivity[1]

- PDMS vs. TangoPlus; same specimen sizes
  - PDMS: $l = 0.08mm$
  - TangoPlus: $l = 1mm$

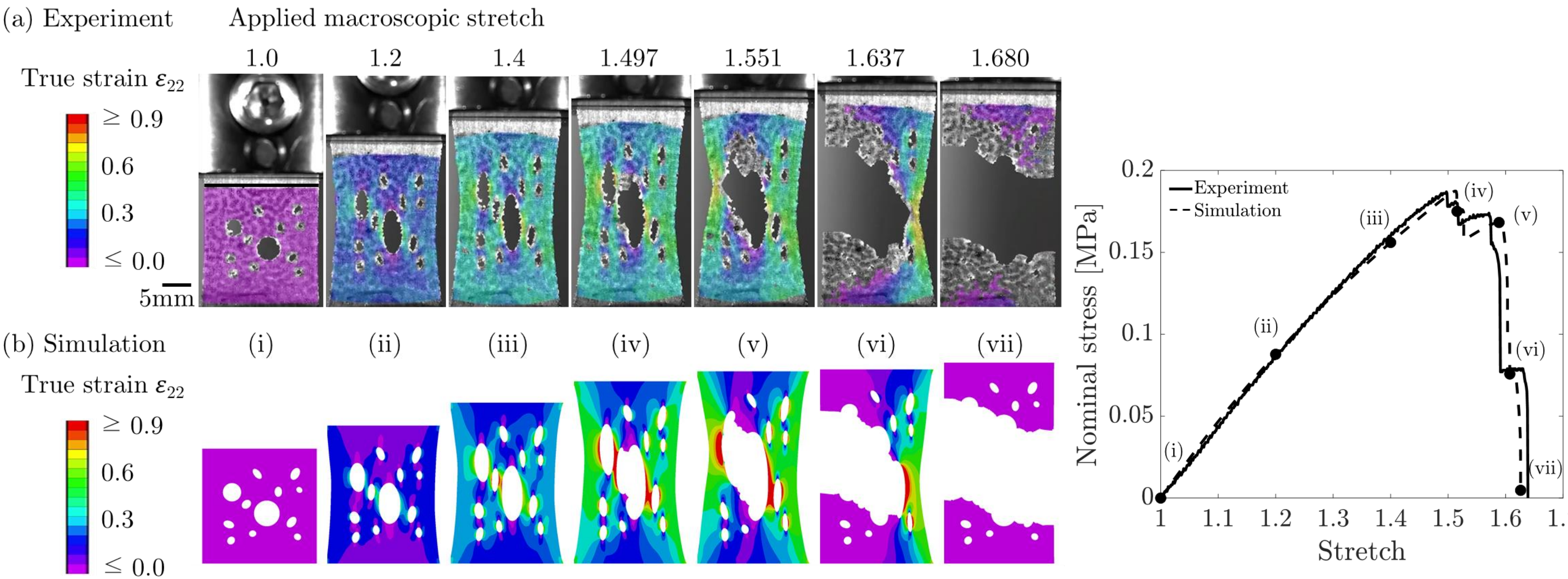
  **More than 10 times**
- $a/l$: 0.5~5 (TangoPlus; $l = 1mm$) → Highly notch length-sensitive
- $a/l$: 5~50 (PDMS; $l = 0.08mm$) → Less notch length-sensitive



# Randomly perforated specimen (TangoPlus)[1]

- Nicely predicted the response **without modification of parameters**
  - Progressive fracture of ligaments



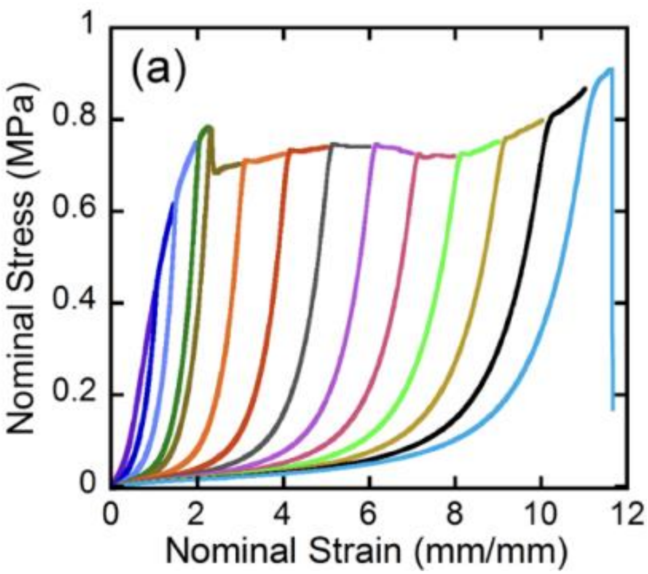
# Conclusion

- **Size-dependent fracture** is clearly observed in experiments[1]
  - Rupture stretch increases as the notch length decreases
  - Size-dependence increases as the notch-root radius decreases
- **The intrinsic length scale** is central to account for the size-dependent behavior[1]
  - The intrinsic length scale $l$ defines the size of diffusive damage zone / fracture process zone
  - The intrinsic length scales were identified from experiments
  - Normalized notch length $(a/l)$ determines the size-dependence
- **Nonlocal continuum model**[2,9] nicely predicted the fracture in elastomers[1]
  - Nonlocal continuum model utilizes experimentally identified intrinsic length scales
  - The model captures the size-dependent fracture in elastomers
  - The model is capable of predicting the fracture of complex geometries

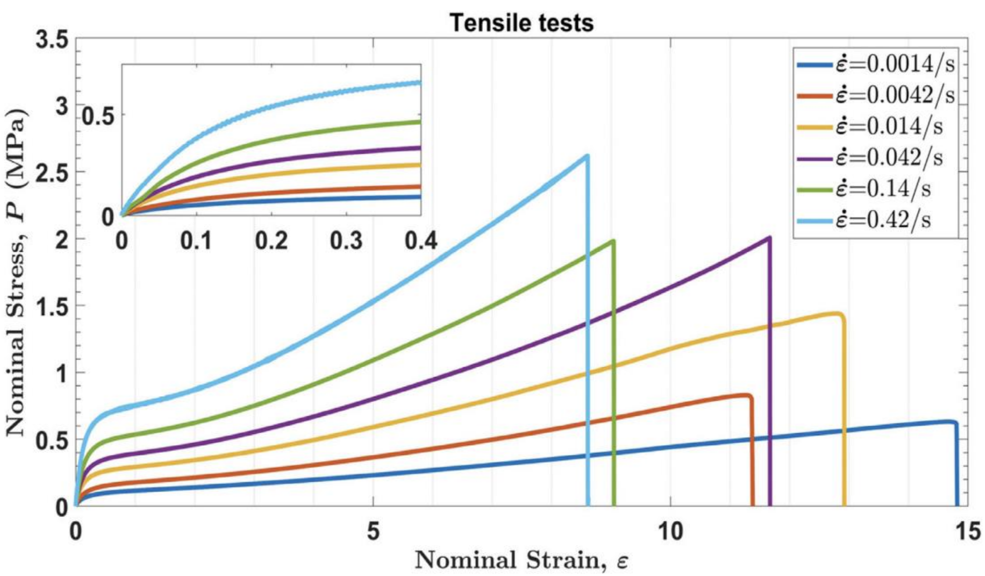
# Future work



- Fracture involving non-trivial dissipation
  - Mullins effect[16-20]
    → Is the fracture behavior influenced by the rate-independent dissipation (e.g., the Mullins effect) ?
  - Viscous dissipation[16-19,21]
    → How to describe complicated deformation and fracture behaviors in polymers?




a) Fracture in double-network elastomers; the Mullins effect and fracture occur[20]




b) Rate-dependent deformation and fracture behaviors in a hydrogel (polyampholyte gel)[21]

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